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AN ACCURATE METHOD FOR FD-TD PLANE WAVE PROPAGATION FOR WIDE BAND EXCITATIONS

Jeff T. MacGillivray and David Dietz

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14. ABSTRACT

An accurate and efficient technique is presented for propagating wide band plane waves in a three-dimensional (3-D) finite difference time domain (FD-TD) grid. Commonly, the incident pulse is created via a separate, one dimensional auxiliary field grid and then applied to the 3-D grid. Although this paper focuses on the separate field formalism, the method can be applied to the scattered field formalism as well. As the wave is applied, a 3-D numerical wave is excited and propagates. If the wave number band of the auxiliary field grid is not nearly equal to that of the 3-D numerical wave, error accumulates resulting in inaccurate results. However, by reconstructing the auxiliary field grid, its wave number band can be closely matched to that of the 3-D numerical wave yielding a much higher fidelity propagating plane wave.

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An Accurate Method for FD-TD Plane Wave Propagation for Wide Band Excitations

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Abstract

In this paper an accurate and efficient technique is presented for propagating wide band plane waves in a three-dimensional (3-D) finite difference time domain (FD-TD) grid. Commonly, the incident pulse is created via a separate, one dimensional (1-D) auxiliary field grid and then applied to the 3-D grid. Although this paper focuses on the separate field formalism, the method can be applied to the scattered field formalism as well. As the wave is applied, a 3-D numerical wave is excited and propagates. If the wave number band of the auxiliary field grid is not nearly equal to that of the 3-D numerical wave, error accumulates resulting in inaccurate results. However, by reconstructing the auxiliary field grid, its wave number band can be closely matched to that of the 3-D numerical wave yielding a much higher fidelity propagating plane wave.

I. INTRODUCTION

With the advent of much faster computer CPU's, larger processor main memory and massively parallel machines, it is now possible to computationally model electrically large media such as building environments, with techniques that previously were not practical. Understanding the microwave environment inside a building is of much interest for communications, for example. Features such as reinforcement bars, I-beams, conduit and interior equipment scatter fields creating a challenging task for an analyst to assimilate and produce useful information. It is in this arena that time domain computer modeling techniques can contribute significantly by calculating overall field distributions and areas of high and low energy. Arrays of virtual field sensors record data that can be used to generate animated graphics of electromagnetic fields; this is not possible in general with laboratory measurement data. Recent research into building microwave environments at the Air Force Research Lab (AFRL) has driven the need to model plane wave propagation of wide band pulses through electrically large meshes. This need has stimulated modeling research to investigate numerical plane wave travel over many computational cells. One modeling technique that in recent years has become one of the most popular for solving Maxwell's equations is the Finite Difference Time Domain (FD-TD) method [1]-[3]. FD-TD is a simple, robust and easy to implement technique for solving time-dependent Maxwell equations. When propagating a plane wave through an FD-TD grid, there are three commonly used algorithms: the total-field, scattered-field, and separate-field formulations. Each technique has strengths and weaknesses and the merits of each are discussed in [2-4]. All techniques, however, have difficulty with plane waves traveling over many cells. In this paper, a new scheme in conjunction with the separate-field formulation is derived and performance results are given.

II. WAVE NUMBER ERROR

All numerical solvers have inherent errors and one common known to FD-TD is numerical dispersion, a result of phase or wave number error. The magnitude of the wave number error depends upon how well the wavelength is spatially resolved and direction of travel through the grid. This error may or may not be significant depending on the electrical size of the mesh and the number of wave transits. Recent work by Nehrbass, Jevtic and Lee [5] discusses a technique to reduce this error however, another related error arises when implementing plane waves which is the focus of this paper.

A plane wave by definition should have spatially constant field values within all planes normal to the propagation direction. When the incident wave is applied to the grid, a 3-D numerical plane wave is excited and propagates through the grid. If at each frequency, throughout the entire bandwidth the applied pulse and the numerical wave traveling through the grid do not have identical wave numbers, then the intended plane wave will not have planes of constant field value. This difference creates fictitious sources along the total/scattered field boundary. In many cases, the error grows large enough to corrupt the entire grid volume yielding inaccurate results.

For propagation directions parallel to one of the grid axes, this error is easy to rectify by using interpolation look-up data, first proposed by Tavflove [3]. This scheme simply interpolates from an array of E- and H-field values obtained from an auxiliary one-dimensional FD-TD grid. Thus, the wave applied to the total/scattered field boundary can be exactly replicated when propagated parallel to one of the axis. In this paper, this technique will be referred to as the field array (FA) scheme. In any two dimensional diagonal grid direction, the dispersion error is small making it simple to use the closed-form incident field (CFIF) scheme. This method applies the field values to the grid via an analytical equation. However, all other propagation directions are not as easy to handle with either scheme and may generate large errors after propagation through many cells. Some recent work by Oğuz, Gürel and Arikan [6] greatly reduces the error by filtering out high frequency noise from the field array and increasing the spatial resolution of the FA equation. Their work shows examples of three dimensional plane waves traveling in a two-dimensional diagonal direction ($k_x=k_y$, $k_z=0$) and primarily focuses on a single frequency excitation. Instead, this paper develops a method for applying a wide band pulse plane wave. The FA equation is reconstructed with a modified cell size, permittivity and permeability that produces a close approximation to the 3-D numerical wave number band over the entire pulse width. For the rest of this paper, this new scheme will be called the wide band field array method (WBFA). Results demonstrate a higher fidelity plane wave that maintains planes of near constant field values normal to its direction of propagation.

III. METHOD DERIVATION

This section first briefly discusses the dispersion equation and then derives two different techniques for optimizing the FA equation for a single frequency plane wave traveling in any given direction. Each method equalizes the wave numbers of the 1-D auxiliary grid and the 3-D grid waves. Then, by combining these two methods via an iterative algorithm, the FA equation is further optimized to simulate a band of wave numbers closely matching those of the 3-D numerically propagating wave. For a given direction, the wave number error varies almost linearly with the number of cells per wavelength as long as the shortest wavelength is sufficiently spatially resolved. Thus a second order equation can be used to match a band of frequencies.

First consider the transcendental form of the dispersion equation for a uniform mesh and plane wave traveling in a given direction defined by θ and φ:

$$\frac{(\Delta x)^2}{(\Delta t)^2 v^2}\sin^2(\frac{\omega\Delta t}{2}) = \sin^2(\frac{\Delta x}{2}k\sin\theta\cos\varphi) + \sin^2(\frac{\Delta x}{2}k\sin\theta\sin\varphi) + \sin^2(\frac{\Delta x}{2}k\cos\theta) \quad . \qquad (3.1)$$

where $\Delta x = \Delta y = \Delta z$ and $v_c = \dfrac{1}{\sqrt{\varepsilon_0\mu_0}}$.

For a given frequency, cell size and time step satisfying the Courant stability limit, there is a resulting numerical wave number that satisfies the equation above. This number can easily be found by an iterative guessing method such as Newton's method. Unfortunately, the numerical wave speed ω/k is not necessarily equal to the wave speed v_c as it ideally should be. There are certain conditions for which they are almost equal, but in general the numerical wave number differs from the exact, analytical k that will produce the correct wave velocity [2, 3].

Now consider the 1-D dispersion equation that applies to the FA equation and let the input variables be the cell size and wave number found from the 3-D dispersion equation above:

$$\frac{(\Delta x)^2}{(\Delta t)^2 v_{1D}^2}\sin^2(\frac{\omega\Delta t}{2}) = \sin^2(\frac{\Delta x}{2}k) \quad . \qquad (3.2)$$

Solving for the speed v yields

$$v_{1D} = \sqrt{\frac{(\Delta x)^2 \sin^2(\frac{\omega \Delta t}{2})}{(\Delta t)^2 \sin^2(\frac{\Delta x}{2} k)}} = \frac{1}{\sqrt{\varepsilon_{1D}\mu_{1D}}} \,. \tag{3.3}$$

ε_{1D} and μ_{1D} of the FA equation can be adjusted so the wave number matches the 3-D numerical plane wave number found in equation 3.1. Note that ε_{1D} and μ_{1D} should give the medium, (usually free space) wave impedance, $Z = \sqrt{\mu_0/\varepsilon_0} = \sqrt{\varepsilon_{1D}/\mu_{1D}}$. By implementing the new ε_{1D} and μ_{1D}, the FA equation can generate the same wave number as the 3-D numerical plane wave for a single frequency yielding a much improved plane wave.

A second method for matching the wave number is as follows. Let the input variables be the wave number and speed found in equation 3.1 and solve for the cell size in equation 3.2 using an iterative method. Again, the wave number of the FA equation matches that of the 3-D numerical plane wave for a single frequency. Both methods can apply the field to the total/scattered field boundary via interpolation from the FA equation.

Up to this point, the FA equation wave number is set equal to the 3-D wave number for a single frequency using one of the two methods above. However, a band of wave numbers needs to be matched to accurately propagate a pulse. This task is a little more complex. Nonetheless, by combining the two methods above it is possible to match an entire band of wave numbers with a high degree of accuracy by employing the algorithm shown below. The wave number at the upper and lower frequencies of the excitation pulse is matched by adjusting the wave speed v_{1D} and the cell size Δx_{1D} of the FA equation. The scheme is as follows:

$k_{\min}$ = Newton iteration of 3-D dispersion equation ($\Delta x, v_c, \omega_{\min}$) = wave number at lower frequency limit
$k_{\max}$ = Newton iteration of 3-D dispersion equation ($\Delta x, v_c, \omega_{\max}$) = wave number at upper frequency limit

Δx_{1D} = guess

$$v_{1D} = \sqrt{\frac{\Delta x_{1D}^2 \sin^2(\frac{\omega_{\max} \Delta t}{2})}{\Delta t^2 \sin^2(\frac{\Delta x}{2} k_{\max})}} \quad \text{(from equation 3.3)}$$

$k_{\min,1D}$ = Newton iteration of 1-D dispersion equation($\Delta x_{1D}, v_{1D}, \omega_{\min}$)

If $k_{\min,1D} \neq k_{\min}$

First, the wave number limits are found from the 3-D dispersion equation. Since FD-TD commonly uses ten cells per wavelength at the highest frequency, this is usually a good upper frequency limit. The lower frequency can be any number less than the upper limit but greater than zero to avoid a divide by zero error in the Newton iteration. The first guess for the cell size Δx_{1D} is the 3-D grid cell size. Next, the FA equation wave speed v_{1D} is found directly from equation 3.3. Now that the upper limit wave number is matched, the lower limit wave number is evaluated via Newton's iteration method. If this wave number is not equal to the lower limit wave number of the 3-D dispersion equation, a new cell size is approximated and the loop continues until both the upper and lower wave numbers match. Successive cell sizes can be approximated by interpolation of the previous two guesses and the corresponding wave numbers $k_{\min,1D}$. The wave number variation from the lower to upper frequency is nearly linear for both the 1-D and 3-D equations thereby producing very closely matched wave numbers and phase errors throughout the frequency range. Since the cell size and time step of the WBFA equation differ from those of the 3-D grid, the field values must be interpolated precisely correctly; otherwise, the field will not be applied accurately

but instead will produce large errors. A nice feature of this method is the efficiency. Since this loop is executed during preprocessing, the run time penalties are negligible.

IV. SINC FUNCTION EXCITATION RESULTS

To measure the accuracy of the WBFA method, a series of plane waves were launched at different angles of incidence. In order to attain an accurate measurement of the error, an empty volume with no scattering objects was used. The 3-D computational domain consisted of 100 X 100 X 100 Yee cells encompassed by 8-cell-thick perfectly matched layers (PML) with a normal reflection coefficient R(0) of 10^{-4} and parabolic conductivity profile. The cell size was chosen at 0.625cm and the time step was selected to be just under the Courant stability limit at 11.92 ps. The separate field formulation was employed with the total field occupying a volume of 88 X 88 X 88 cells and, accordingly, a six cell thick scattered field region The input pulse was a sinc function with a frequency band from DC to 4.8GHz. Ten cells per wavelength resolved the highest frequency. The peek amplitude of the sinc wave was one V/m and entered the volume at time step 500. All tests were run on 360MHz SPARC Ultra 60's.

The same permittivity and permeability values for the 3-D FD-TD equations were implemented for all runs. No optimization for one propagation direction was performed. Instead, an optimized permittivity and permeability were computed via a weighted average of the 3-D dispersion equation over all angles of incidence. This was applied using a scheme derived by Nehrbass, Jevtic and Lee which greatly decreases the average wave number error over all directions [5]. These same values of permittivity and permeability were used in the 3-D dispersion equation (Eq. 3.1).

In addition, all test runs were performed with the wave number of the FA equation matched to the 3-D wave number at 4.8GHz by adjusting the permittivity and permeability of the FA equation. The higher the frequency, the lower the spatial wave number resolution and the greater the wave number error. Thus, the wave number was matched at the frequency of highest wave number error.

Ideally, the E-field values in the total field zone should be exactly equal to those of the incident pulse of the FA equation and the fields in the scattered field zone should be exactly zero. However, there will be error due to the nature of finite difference numerical methods. The results shown in Figs. 1-4 give the maximum E-field error over both the total and scattered field zones. The error is calculated by first taking the difference of the 3-D numerical E-field value and the corresponding E-field value on the 1-D auxiliary equation. Then, this difference is normalized to the maximum amplitude of the input plane wave (1 V/m). In each figure the test results for the FA scheme are in part (a) and the test results for the WBFA scheme are in part (b).

The first test run launched the plane wave in the xy-direction ($\theta = 90^o$ and $\varphi = 45^o$) with the E field vector pointing in the z-direction. The error results are shown in Figure 1. Initially, the error rises and then drops after about 250 time steps. The initial excitation of the wave causes high frequency noise which falls off after it passes. Then the error rises again starting at about 500 time steps and falls at about 750 time steps. At 500 time steps the peak of the sinc wave enters the grid. This peak rises far above the previous preceding pulse and therefore has the same effect as the initial excitation. In addition, the amplitude is larger and therefore simply has a larger error. As shown, the WBFA scheme results in an error about three orders of magnitude less than the FA scheme.

In the second and third test runs, the E field vector has nonzero values in all three directions. The angle of incidence for run two is $\theta = 45^o$ and $\varphi = 45^o$. In this case the E field vector is equal in magnitude in all three components and the peak error of the WBFA method is about two orders of magnitude below the FA method (Fig. 2). In Figure 3, the angle of incidence is $\theta = 22.5^o$ and $\varphi = 22.5^o$. In this case the E field vector has different magnitudes in all three components and the peak error of the WBFA method is more than an order of magnitude below that of the FA method.

Figure 4 shows the results of the final run with an angle of incidence at $\theta = 90^o$ and $\varphi = 0^o$. As explained before, in this case the FA equation can replicate the 3-D numerical traveling wave. Here, the optimal cell size, permittivity and permeability are equal respectively to those of the 3-D grid solver and the FA equation yields nearly identical results. The resulting error is due to the interpolation from the WBFA or FA equation as it is applied to the 3-D grid.

The FA and WBFA equation accuracies depend on the propagation direction. Both methods have highest accuracy along the x-axis and decrease in accuracy as the propagation direction is changed. However, the FA

equation's error rapidly grows to as high as 10^{-1} (to one decimal place at an angle of $\theta = 90^o$ and $\varphi = 45^o$. On the other hand, the WBFA method attains maximum error of 10^{-2} at an angle of $\theta = 22.5^o$ and $\varphi = 22.5^o$.

V. CONCLUSIONS

In the test runs performed, the WBFA scheme had a maximum error of 10^{-3} for a sinc pulse containing a frequency band from zero up to 4.8GHz at ten cells per wavelength. For many applications such as field distributions in building environments, this is more than acceptable accuracy. The error appears to be greater in directions where the E field components differ in magnitude and are not equal to zero. Conversely, the FA method has errors in the first decimal place.

Accurate propagation of wide band plane waves is one of the obvious advantages of the WBFA method. Time domain methods such as FD-TD are often utilized for pulse propagation. Pulses are especially convenient for coupling problems since the input pulse and the coupling points of interest can be transformed to the frequency domain allowing for an entire frequency band of coupling results with one code run.

Although this scheme was applied to the separate-field formulation, the same WBFA equation could be applied to the scattered-field formulation as well. By matching the wave number band of the WBFA equation with that of the 3-D grid, the subtraction noise should be reduced significantly. This is especially important for small coupling problems.

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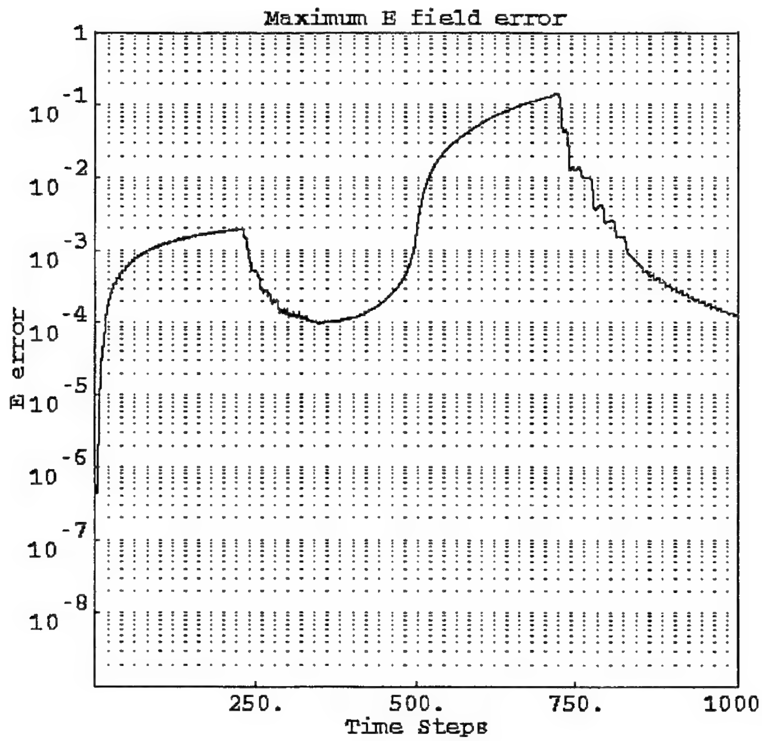

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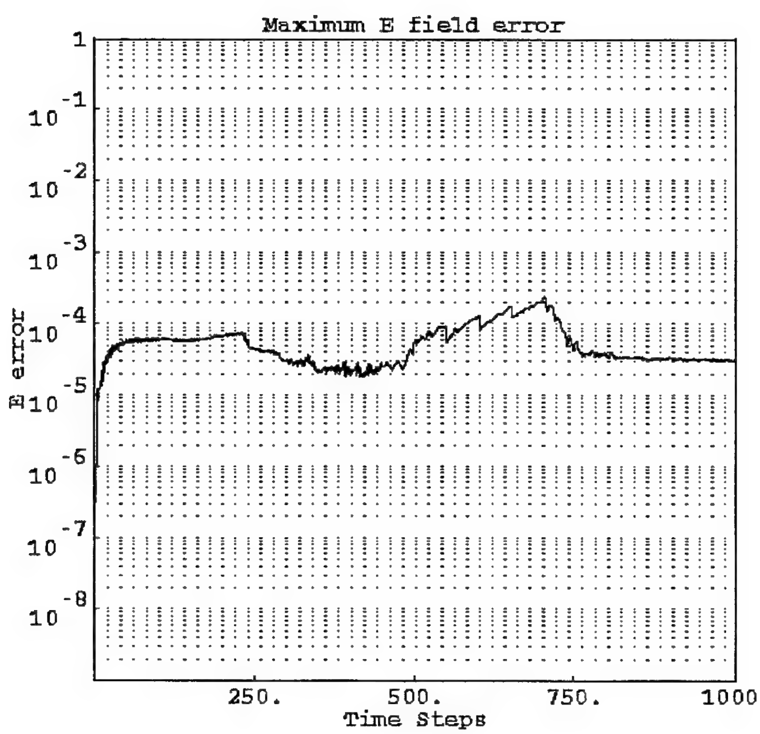
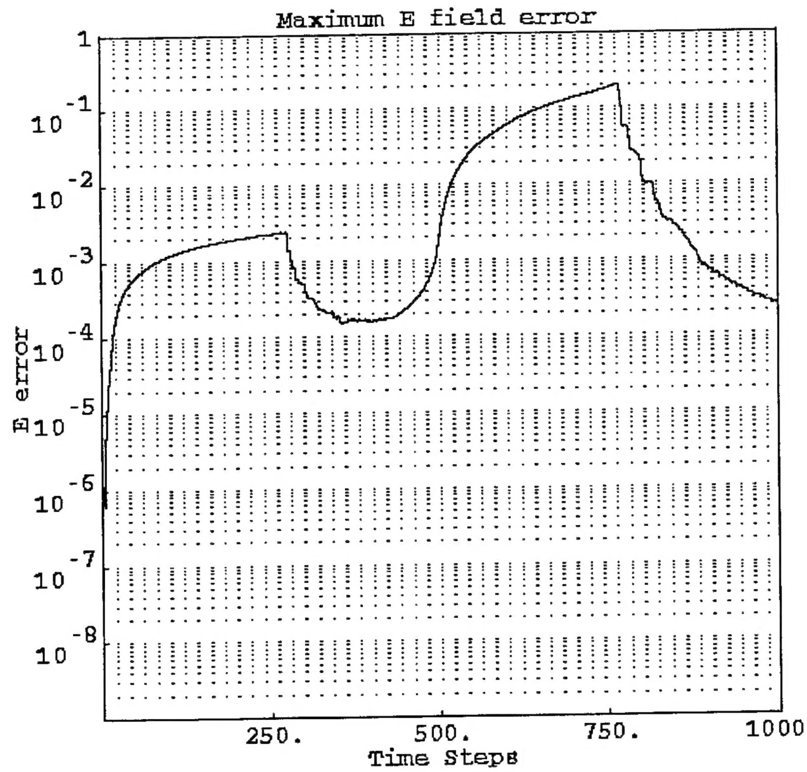


(b)

Fig. 1. Error results for plane wave incident at $\theta = 90^o, \varphi = 45^o, Polar = 90^o$. (a) FA scheme. (b) WBFA scheme.



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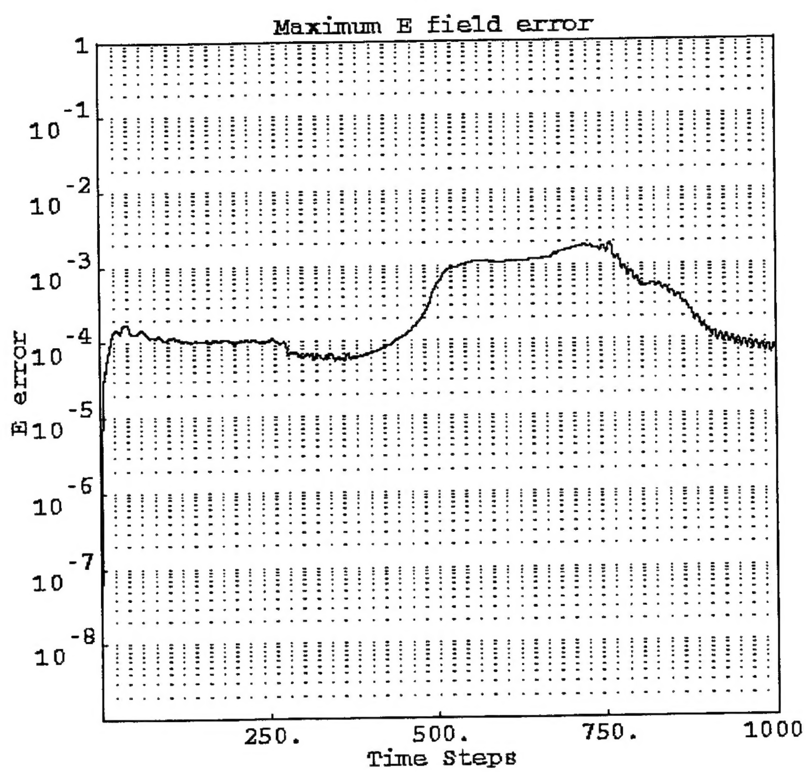

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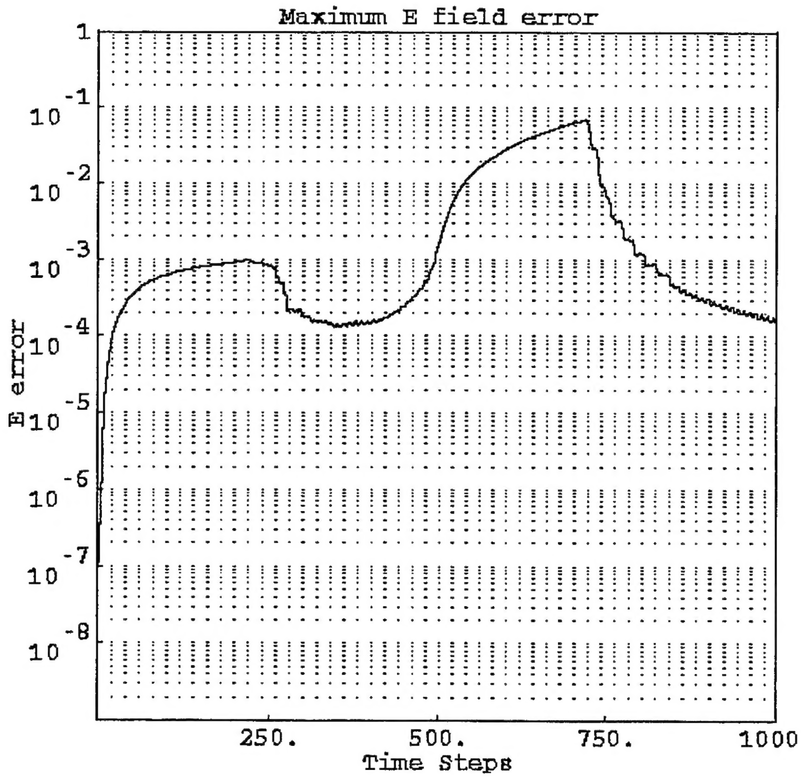
Fig. 2. Error results for plane wave incident at $\theta = 45^o, \varphi = 45^o, Polar = 90^o$. (a) FA scheme. (b) WBFA scheme.



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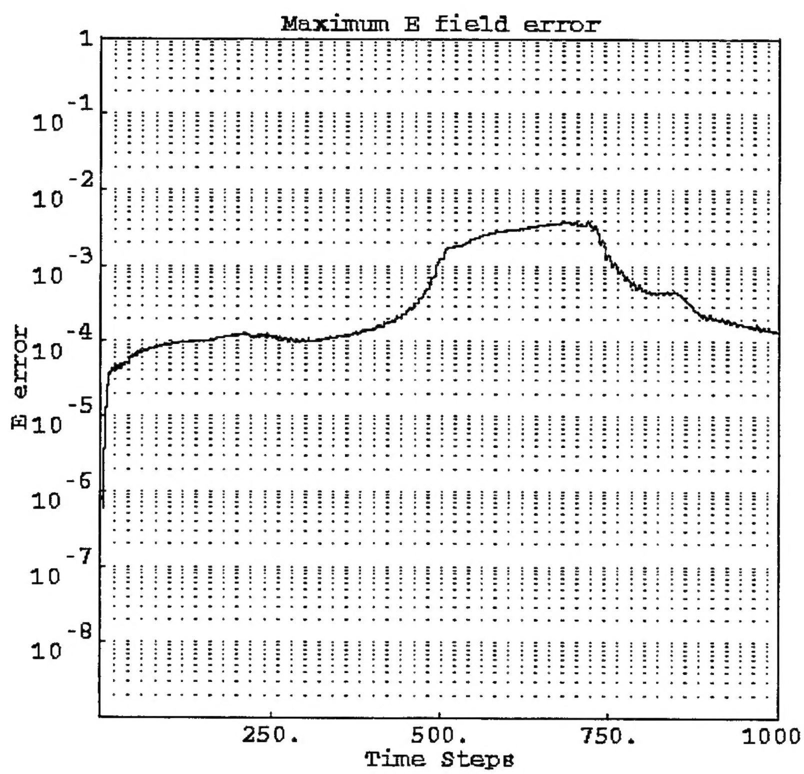
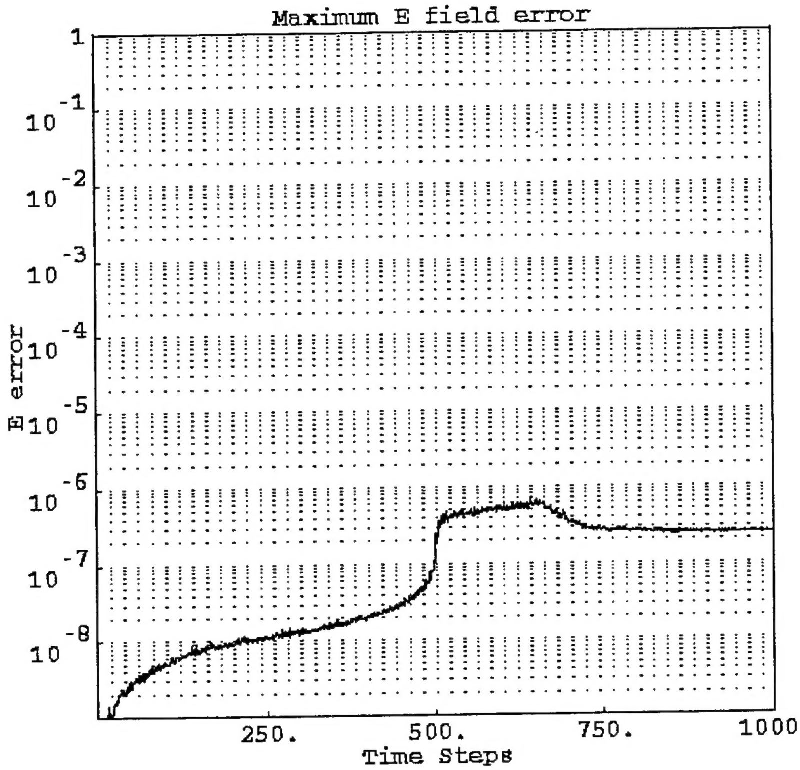


(b)

Fig. 3. Error results for plane wave incident at $\theta = 22.5^o, \varphi = 22.5^o, Polar = 90^o$. (a) FA scheme. (b) WBFA scheme.



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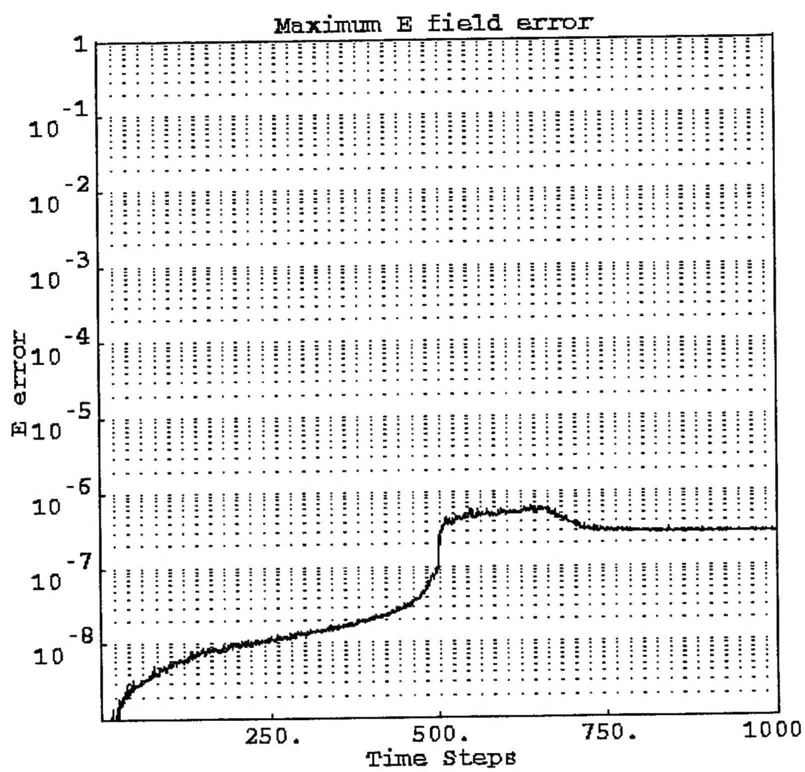

(b)

Fig. 4. Error results for plane wave incident at $\theta = 90^o, \varphi = 0^o, Polar = 90^o$. (a) FA scheme. (b) WBFA scheme.

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